UNLOCKING THE VALUE OF SPECTRUM USING OPTIMIZATION TOOLS

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The radio spectrum repacking designed by the FCC in the recently completed Broadcast Incentive Auction (BIA) is innovative and groundbreaking in its implementation of auction procedures, but also in terms of the development of cutting-edge tools in interference modeling and spectrum management. The techniques used in the BIA have potential for broader use in spectrum management, especially for regional band planning.

This paper provides a high-level vision of innovation in spectrum planning. It elucidates general optimization techniques that have wide applicability in spectrum management, using repacking in the BIA as an illustration, and indicates how these techniques can form the basis for new and innovative spectrum management tools for regulators and network operators.

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INTRODUCTION

Demand for mobile services has, in a word, exploded. In almost every corner of the globe, people are demanding more types of services using mobile technologies to gain mobility and convenience. In 2017, there are more than 4.7 billion unique mobile subscribers, with over 7.6 billion mobile connections. Some global citizens, particularly in the developing world, often have more than one subscription.[1] The developing world is driving much of this growth and is forecast to contribute more than 90% of the incremental subscription growth over the next three years.[2] In addition to the demand for mobile services caused by the sheer number of subscribers, increasing numbers of users expect their devices to provide more and more services, with functions ranging

1. *Mobile Economy 2017*, GSMA, http://www.gsma.com/mobileeconomy/ [https://perma.cc/FUA8-B4H8] (last visited Apr. 15, 2017).
2. *Id.*

from making simple voice calls, to sending data files, to browsing the internet, or watching real-time video.

Clearly, this increased demand for mobile service translates into increased demand for its most essential, most fundamental input: spectrum. This great need for the resource means that it must be managed efficiently; spectrum must be allocated and utilized in ways that consumers, and the societies where they live, value most. Given the dynamism of the mobile marketplace, this goal can be furthered by allowing for flexibility in terms of the services and technologies that can be offered or operated in the spectrum space.

Such flexibility is in tension with the basic reason for spectrum management in the first place: controlling interference. No two users can utilize the same frequency in the same geographic area at the same time without causing interference to one another (absent sophisticated and costly receivers that can differentiate between wanted and unwanted signals).[3] Due to the nature of radio waves, users often cannot even use the same frequency in adjacent areas or frequencies in the same area without interference issues arising. It is the hallmark objective of spectrum management to allow for use of this important resource while at the same time controlling interference.

Until a few decades ago, spectrum was abundant and government spectrum managers allotted and assigned large amounts of spectrum for broadcast television and satellite services as well as to government agencies.[4] Spectrum that has been previously assigned and is not currently being used in an efficient manner, or even at all, can be very difficult to reassign. However, with increasing demand for mobile services, spectrum managers are revisiting spectrum allocations and assignments in hopes of accessing more spectrum to support new technologies, such as fifth-generation wireless broadband technologies ("5G").[5]

When seeking to reassign spectrum, public policy makers must consider moving or realigning existing services to use different spectrum bands or less spectrum in existing bands. In the case of relocating a service to a different band, the regulator must identify the new band and possibly deal with existing assignments there, and then determine a transition plan for the move. When

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3. *See, e.g.*, *RF Filter Basics Tutorial*, RADIO-ELECTRONICS.COM, http://www.radio-electronics.com/info/rf-technology-design/rf-filters/rf-filter-basics-tutorial.php [HTTPS://PERMA.CC/SB32-J2AP] (last visited Apr. 15, 2017).

4. *See, e.g.*, *United States Frequency Allocations*, NTIA (2003), https://www.ntia.doc.gov/files/ntia/publications/2003-allochrt.pdf [https://perma.cc/XL2X-R4GL].

5. *5G Spectrum Policy Position*, GSM ASS'N, (Feb. 2, 2017), http://www.gsma.com/spectrum/5g-spectrum-policy-position/ [https://perma.cc/U9HU-NGQE].

"repacking" or "re-planning" a band with existing users to free up spectrum for other uses and users, regulators must consider how to reassign licenses to the existing users in a manner that frees up spectrum for new users while respecting non-interference rights of existing licensees. This planning can be complicated, as moving one licensee will create a daisy chain effect that requires considering the impact on all adjacent licensees, both in terms of spectrum and geography. A common example of repacking a band is the "digital dividend", the transition from analog to digital television where a new set of digital channel assignments is determined for broadcasters to replace less efficient analogue technology.[6]

The United States completed its digital dividend in 2009 and has now undertaken a bold new approach to repack its TV bands known as the Broadcast Incentive Auction ("BIA" or "auction").[7] The BIA has a goal of clearing spectrum in the upper portion of the UHF television band to be repurposed for mobile broadband services and was inspired by early spectrum management discourse at the FCC[8], but gained traction with the 2010 U.S. Broadband Plan[9]. Early studies of the efficiency of digital TV channel assignments in the TV bands showed that by simply repacking existing broadcasters in a more efficient manner while still respecting existing interference protections, the FCC would be able to free up to 36 MHz of spectrum (6 digital broadcast TV channels).[10] This fell short of the Broadband Plan's goal to identify 500 MHz more spectrum for mobile uses.[11] In order to free up TV band spectrum for mobile, some broadcasters would need to relinquish some of their spectrum usage rights. In 2012, Congress granted the FCC the authority to offer broadcasters incentive payments to relinquish some or all of their spectrum usage rights. This gave the FCC the ability to design and conduct the first-ever Broadcast Incentive Auction.[12]

6. *The Digital Dividend*, INT'L TELECOMM. UNION, http://www.itu.int/net/itunews/issues/2010/01/27.aspx [HTTPS://PERMA.CC/465U-8JDJ] (last visited Apr. 19, 2017).

7. *Broadcast Incentive Auction*, FCC, https://www.fcc.gov/about-fcc/fcc-initiatives/incentive-auctions#block-menu-block-4 (last updated Apr. 13, 2017) [HTTPS://PERMA.CC/D5BZ-3SJP].

8. Evan Kwerel & John Williams, *A Proposal for a Rapid Transition to Market Allocation of Spectrum*, (FCC, OPP Working Paper No. 38, Nov. 2002), http://wireless.fcc.gov/auctions/conferences/combin2003/papers/masterevanjohn.pdf [https://perma.cc/X33R-2Q6F].

9. FCC, CONNECTING AMERICA: THE NATIONAL BROADBAND PLAN (2010), https://www.fcc.gov/general/national-broadband-plan [hereinafter BROADBAND PLAN]. [https://perma.cc/ZSN2-SWKN].

10. *Id.* at 89.

11. *Id.* at 10.

12. Broadcasters were given the option of turning back their entire 6 MHz channel to go off the air, turn back the channel but remain on the air by sharing a channel with one or more other broadcasters, or move from a UHF channel to a VHF channel, subject to availability. *See* Middle Class Tax Relief and Job Creation Act of 2012, Pub. L. No.

The FCC's resulting BIA design is a complex and comprehensive approach to addressing the goal of repurposing part of the TV band for mobile use.[13] The auction process involves a number of innovations worthy of study to those interested in modern spectrum management, including designing a two-sided spectrum auction (with mobile operators as buyers and broadcasters as sellers), creating novel auction rules for a descending ("reverse") auction for broadcasters and an ascending ("forward") auction for mobile operators, and the lynchpin, the creation of an approach for repacking the band by determining feasible TV channel assignments for broadcasters in real-time during the auction followed by a more comprehensive final channel assignment process.

Regarding the repacking element, the approach developed by the FCC involves several interesting elements that could be useful beyond the BIA. Specifically the auction:

1) takes advantage of advanced math and computing to solve complex spectrum assignment problems very quickly;
2) is agnostic to the type of service(s) being considered as it is based on translating technical data from propagation models into forms that optimization software can make use of;
3) accommodates political borders or boundaries as long as that data can be factored into the propagation models; and
4) can be used as a scenario analysis tool to explore "what if" scenarios and facilitate evidence-based policy decision making.

Because of the BIA's promise in future band planning and re-planning, this article will focus on the repacking aspect of the reverse auction to explore advances in computing and mathematical optimization that may prove useful to studying and solving spectrum management challenges in other bands for other services. While the approaches described in this article were developed to address the very specific problem the FCC faced, it will become apparent that the strategies have the potential to be used outside of the BIA context, indeed outside the context of an auction at all, and apply to band planning and re-planning in other countries and even across regions.

This article is organized into eight parts. Part 1 is a primer on the BIA; Part 2 describes how mathematical optimization

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112-96, §§ 6402, 6403, 125 Stat. 156 (2012) [hereinafter Spectrum Act].

13. *See How it Works: The Incentive Auction Explained*, FCC, https://www.fcc.gov/about-fcc/fcc-initiatives/incentive-auctions/how-it-works (last updated Feb. 3, 2017) [https://perma.cc/FUT4-X2QC].

techniques are used in the repacking problem; Parts 3–5 provide specific details about the optimization approach; Parts 6–7 discuss the information needed from incumbent services to use the optimization to repack channels; and Part 8 considers other uses for this approach to solve other spectrum management challenges.

I. PRIMER ON THE U.S. BROADCAST INCENTIVE AUCTION

Spectrum auctions have been a part of a regulator's toolkit for spectrum management for more than two decades.[14] In conventional auctions, licenses are assigned solely based on the bids placed in the auction. Things like interference considerations are typically not addressed in the assignment mechanism, and avoiding interference becomes an obligation for the winning bidders, which is addressed in service rules, license terms and conditions, and enforcement processes. Bidders must therefore assess the interference rules and their potential impact on deployment scenarios when determining the value proposition of a given license for their business. The bidders must model the potential impact from/on adjacent licensees and/or other services using propagation modeling software.[15] Propagation modeling is not an exact science. The uncertainties and difficulties in making assessments about potential interference[16] translate into one of the challenges faced by bidders in making spectrum valuations.

Modern spectrum assignment mechanisms, however, can take advantage of advances in computing to incorporate elements of interference modeling into the mechanism itself. The FCC's BIA is an example of a design that uses "spectrum optimization" as part of its auction design. In this case, optimization becomes necessary to maximize the efficient assignment of spectrum because of the incumbent users whose spectrum usage rights must be considered as part of the process. Most spectrum bands currently have incumbent users, and optimization techniques will become more important components in spectrum management in the coming years as regulatory authorities seek to find ways to maximize the use of spectrum. In particular, these techniques will increase in importance in relation to examining how different services can coexist in terms of spectrum and geography.

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14. *See* Thomas W. Hazlett, *Assigning Property Rights to Radio Spectrum Users: Why did FCC License Auctions Take 67 Years?,* J.L. & ECON., Oct. 1998, at 529.

15. A propagation model is a mathematical model that determines how radio waves propagate under varying conditions and environments. The propagation is naturally influenced by the local surroundings of the antenna emitting radio signals, but the propagation also highly dependent on the frequency of the radio waves. There are many different propagation models that cater to different frequency ranges and environments. Among the more popular propagation models are the Longley-Rice model and the Hata Model. For more information on propagation models, *see infra* section 5.

16. *See infra* Section II (B)(4).

In the case of the BIA, the efficient repacking of incumbent broadcasters heavily influences the amount of spectrum that can ultimately be repurposed. The auction can be viewed as a means of gathering information that is needed to solve an optimization problem, which in this case is assigning channels to the broadcasters that will remain on the air (either because they don't participate in the auction or they participate but choose to reject the relinquishment incentive made by the FCC) in a manner that frees up currently used spectrum for new uses.

In the BIA, TV stations are given incentives to relinquish some or all of their spectrum usage rights in exchange for monetary compensation.[17] The relinquished licenses will allow the FCC to repack the remaining broadcasters in order to clear spectrum that can be sold as mobile licenses. The incentive compensation provided to TV stations comes out of the revenue from auctioning the mobile licenses. Revealing the right balance between demand and supply of spectrum is informed by two auctions: a reverse auction, where the compensation for TV stations is determined (price offers to TV stations decrease in each round of bidding), and a forward auction, where mobile licenses are auctioned (here the prices increase in each round). In general, the BIA process will not end until the proceeds from a completed iteration of the forward auction (determined at the point where there is no excess demand for the mobile licenses) exceed the compensation needed from a corresponding completed iteration of the reverse auction. The two auctions are connected by a series of spectrum optimizations whose purpose is to repack the TV stations based on the bids collected in the reverse auction and convert the cleared spectrum to mobile licenses of as high quality as possible.[18] The spectrum optimization in the reverse auction considers new assignments of TV stations to channels, whereas the forward auction for mobile licenses is a variant of a familiar simultaneous multi-round auction.

17. *Expanding the Econ. and Innovation Opportunities of Spectrum Through Incentive Auctions,* GN Dkt. No. 12-268, Report and Order, 29 FCC Rcd. 6567, n.322–349 (2014) [hereinafter BIA Report and Order].

18. Quality, in this case, refers to the amount of interference to mobile licenses from TV stations.

FIGURE 1

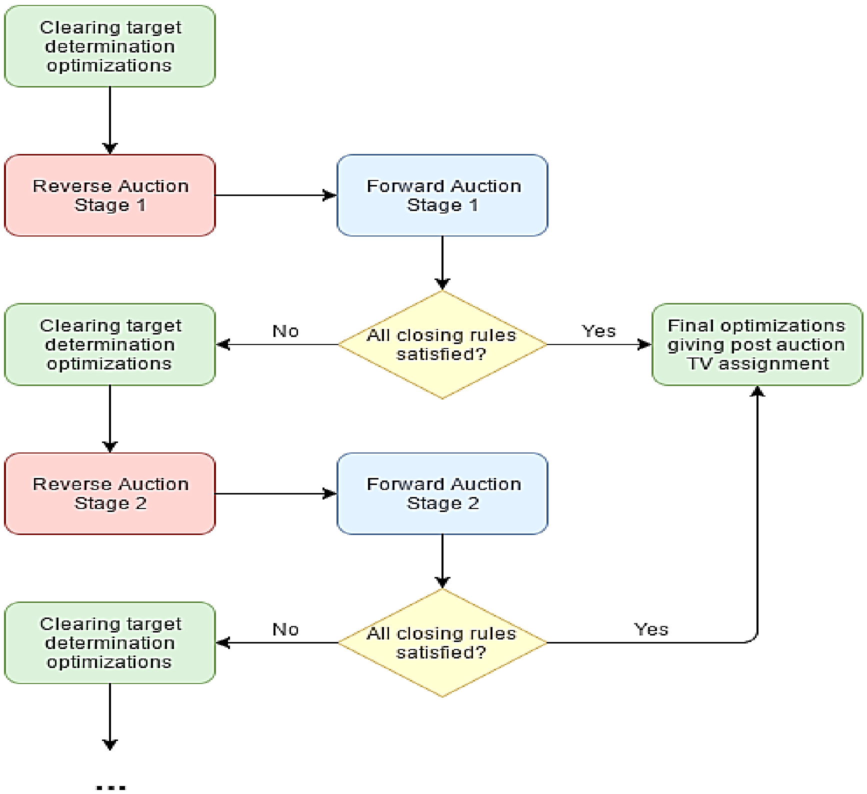

Figure 1: Flow Chart of the U.S. Broadcast Incentive Auction

Because the amount of spectrum cleared depends on broadcaster and mobile operator participation, the BIA is divided into stages that seek to clear decreasing amounts of spectrum (called 'clearing targets'), stopping at the point where mobile operators are willing to pay the amount required to clear broadcasters from a given quantity of spectrum repurposed as mobile licenses. In the first stage, the FCC decides the initial clearing target based on the result of a series of optimizations that consider initial broadcaster commitments to relinquish spectrum.

Once the clearing target is determined—that is, the number of TV channels that will be cleared for mobile use—the reverse auction can begin. During each bidding round, participating bidders (TV station license holders) are offered decreasing compensation to relinquish their spectrum usage rights. Bidders can choose to accept or reject the amount offered. If the bidders reject the offer, they exit the auction and will continue to hold their broadcast license, requiring the FCC to assign them to a channel in the newly configured TV band. If the bidders accept the offer, they remain in the auction and with the offer amount, decreasing in subsequent rounds as long as the FCC determines there is a viable TV channel for that station to occupy in the remaining portion of the TV band.

If the FCC determines there is no longer a viable channel for the bidder's station to occupy, the accepted offer is their final offer for that stage and if the stage succeeds (a successful stage is one in which the forward auction generates sufficient revenue to cover the reverse auction and other costs), the bidders are paid that amount to relinquish their spectrum usage rights. The FCC refers to this as the bidder's price being "frozen" because it will not decrease further in that stage. For stations that are not frozen, the FCC lowers the compensation on an offer through multiple rounds, until a sufficient number of TV stations reject the offers and drop out so that the cleared spectrum matches the target set. The cleared spectrum is then auctioned off in the forward auction. The BIA ends in the stage when the proceeds from the forward auction exceed the compensation requirement from the reverse auction. If this does not happen in the first stage, then the clearing target is lowered and further iterations of reverse and forward bidding are held, tending to remove TV stations that require high compensation along with mobile bidders that are not willing to increase their bids. Generally, the reverse and the forward auction can be seen as information-gathering mechanisms that help determine the value and thus best use of the spectrum.

The details of bidding in the reverse and forward auctions could be the topic of another paper. This article, however, focuses on the underpinning challenges of spectrum optimization that determines whether there are still viable channel assignments for broadcasters in the reverse auction, and how spectrum management policies are reflected in this mechanism. Although the details in this mechanism are specific to the North American TV markets, they illustrate how a new and innovative program of spectrum optimization for license assignment can be designed and delivered in practice, whether utilized in an auction context or just as a part of planning, or re-planning, spectrum assignments in a given band or bands. The BIA is not just an auction with complicated rules; it represents a fundamentally new approach to spectrum management, in which auctions are used as information-gathering mechanisms, which in turn are used to optimize spectrum assignments and therefore efficiency. Moreover, the auction has the benefit of being an approach that has been implemented in practice, not just laid out in theory.

II. OPTIMIZATIONS IN THE U.S. BROADCAST INCENTIVE AUCTION

Optimization relies on constraints and objectives to be defined in a formal and unambiguous way. These definitions reflect the spectrum regulator's general spectrum management policies. For example, for the incumbent users involved, the regulator has the responsibility of assessing the interference impacts arising from its actions in processing bids in the reverse auction, in contrast to conventional auctions, where interference considerations are not part of the winner determination process.

FIGURE 2

> Mathematical optimization is a powerful tool that solves a wide range of problems involving a set of rules that all solutions must satisfy. These rules are called constraints. A solution satisfying all constraints is called feasible. Getting a feasible solution is not always enough; sometimes solutions are needed that meet further objectives defined by an objective function. The distinction between constraints and objective functions is that although both can be rules—constraints *must* be satisfied whereas objective functions should be satisfied as best as possible.
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> A problem solved by mathematical optimization techniques is the graph-coloring problem (Figure 3). Given a set of colors, is it possible to color each node so no neighboring nodes have the same color? The constraints in this case are the number of colors and the fact that no neighboring edges can have the same color. If there is only one color and more than one node in the graph, then no feasible solution exists. Figure 3 shows a feasible solution for the Petersen graph using three colors. In this formulation, there is no objective function. Requiring feasible colorings with as many nodes colored green as possible, or as few nodes colored blue as possible, or using as few different colors as possible, are all examples of objective functions. In the case of coloring the Petersen graph, the minimum number of colors that can be used is three: this is exemplified in Figure 3. Incidentally, Figure 3 is also an example of the minimum number of nodes colored by the same color (3 nodes), and the maximum number of nodes colored by the same color (4 nodes).

In spectrum optimization, policy objectives can appear either as constraints on the feasible solution or as objective functions in the optimizations. There may also be (as in the BIA) sequences of optimizations to reflect the relative importance attached to different policy objectives. The highest priority of all is reflected in the constraints that define the feasible region. In the BIA, the main constraints are for protection of TV coverage areas. Optimization problems can be hard to solve to optimality, which is finding a solution that can be proven to have the best possible objective value. In practice, we need to work within the boundaries of what is possible computationally to get the best solutions possible. This may mean choosing relatively stringent constraints in order to maintain feasibility of the algorithmic implementation. We should also be ready to use 'better' formulations as the opportunities arise. In connection, it is important to make sure that constraints (e.g. from interference) are not overly conservative, as this will prevent unlocking economic value.

The goal of the optimization is to find a new channel for every TV station requesting one in such a way that the new highest TV channel used is lower than the current highest TV channel used, therefore freeing up some of the current TV band for other uses. At the outset, each TV station commits to one or more different relinquishment options or indicates that it is not interested in relinquishing its license for the initial (high) compensation offer, and thus, does not want to participate in the auction. Based on these responses, the FCC decides on a clearing target aided by a series of optimizations. The process of going from TV station responses to deciding a clearing target is guided by policies and is implemented as a series of mathematical optimization models. One of the fundamental policies is that if a TV station does not want to participate in the auction, then it will stay in its current frequency band (not necessarily on its current channel), which is one of UHF (TV channels 14 and higher), High-VHF (TV channels 7-13), or Low-VHF (TV channels 2-6).

A. Repacking Essentials

Repacking incumbents is the key to spectrum optimization, and for the BIA it is repacking of the TV band. Repacking can be done for any spectrum band and does not require a reverse auction such as in the BIA. A reverse auction frees up more spectrum than repacking could do on its own, as broadcasters in the reverse auction are participating to consider relinquishing some or all of their spectrum usage rights. In this section, and the following, repacking of the TV band is used as an example to illustrate more generally spectrum repacking and the interplay between TV stations and the mobile networks as an example of spectrum sharing.

FIGURE 3

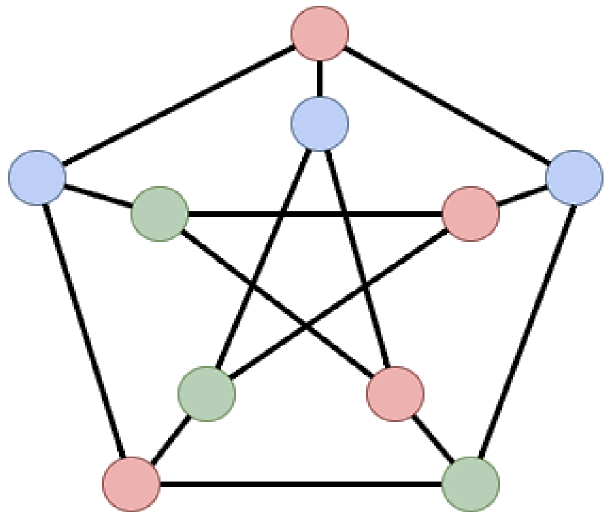

Figure 3: A graph and its coloring using only three different colors. (This graph is called the Petersen graph and it should not be difficult to convince oneself that two colors will not suffice.)

Generating new station-to-channel assignments can be considered mathematically as a graph-coloring problem. The vertices of the graph are TV stations and edges between TV stations indicate that they interfere with each other if assigned the same channel. The problem of finding a new station-to-channel assignment is then to color the vertices of the graph such that no neighboring vertices are colored the same (the color represents the broadcast channel for the node, i.e. the TV station). The problem is easily solvable if we allow each vertex to have a different color, but that is the equivalent of assigning each station to a unique channel. This is a highly inefficient use of the spectrum. The frequencies can be reused as long as there is enough distance between the transmitters. Instead of using this inefficient model, the number of channels available can be limited. Solving this problem is highly dependent on the graph and the number of channels allowed (and if the number of channels is too low then there will be no solution). Graph coloring falls into the category of mathematical problems that are so-called NP-complete.[19] This essentially means that there is no efficient algorithm to solve the problem. For some graphs, the graph coloring problem is easy. For others it is very difficult.

19. The problem of coloring a graph with a specified set of colors is NP-complete. *See* Michael Garey, David Johnson, & Larry Stockmeyer, *Some Simplified NP-Complete Graph Problems*, 1 THEORETICAL COMPUT. SCI. 237 (1976).

The graph used in the optimizations during the United States BIA is visually very complicated because TV stations operate at high power and their signals travel far, causing interference over great distances (the greatest distance between two interfering TV stations is 420 km). The number of colors is initially limited by no stations allowed above channel 51. The graph in the BIA has 2,900 nodes—one for each TV station in United States and Canada. Each node is on average connected to thirty-four other nodes. Therefore, each TV station will on average interfere—across all channels—with 34 other TV stations.

It is important to recognize that there are regional differences for how tightly the TV band can be packed. In urban areas where there are more TV stations, it will be harder to find channels for all of them. Consequently, the FCC may need to allow some TV stations to broadcast on channels in the new mobile band and share the spectrum with mobile users.[20] Determining these stations and their channels will be an essential part of the optimizations.

FIGURE 4

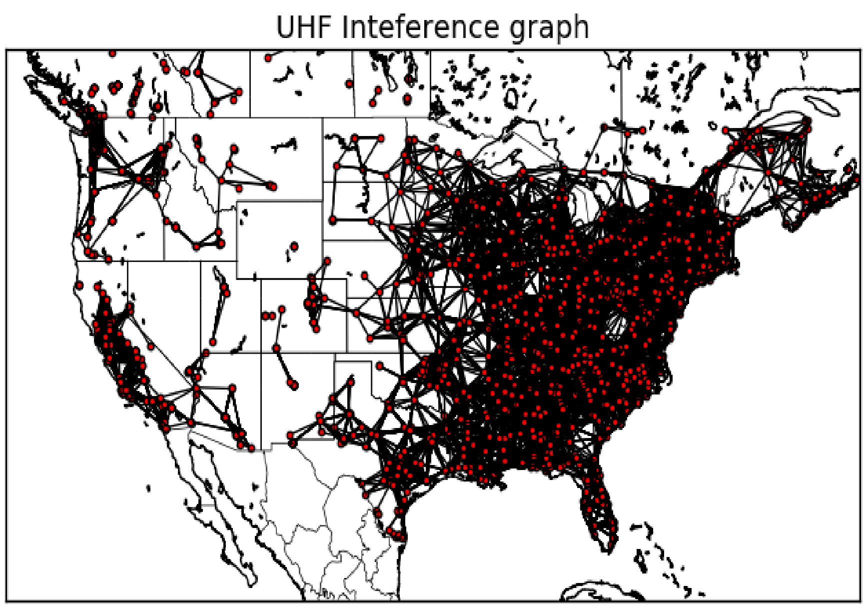

Figure 4: Red nodes are TV stations and there is an edge between two nodes if there exists channels where the TV stations would interfere with each other.

20. The FCC calls the case where a broadcaster is assigned to a channel that overlaps with the otherwise cleared spectrum an "impairment." *Application Procedure of Broadcast Incentive Auction Scheduled to Begin on March 29, 2016, Technical Formulas for Competitive Bidding*, GN Dkt. No. 12-268, Public Notice (released Oct. 15, 2015).

First, to run the repacking a band plan must be chosen. This essentially specifies the cut-off between the new TV band and the mobile licenses. Each possible clearing target has an associated band plan.

FIGURE 5

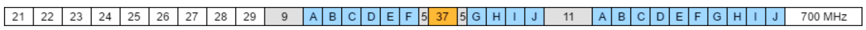

Figure 5: The band plan for the 126 MHz clearing target (the initial clearing target for Stage 1 of the BIA, which subsequently failed). White blocks on the left are TV channels of 6 MHz each and blue are mobile blocks of 5 MHz each. The grey blocks are guard bands. Channel 37 is protected for radio astronomy and wireless medical telemetry use.

A clearing target is chosen by optimizing the spectrum for each of the band plans and choosing the one with the best station-to-channel assignment, where "best" is determined by the spectrum management policies that have been adopted.

B. Objective Functions

Usually a regulator wants to enforce and achieve several different policies and goals in an optimization to get a new station-to-channel assignment, such as protection of incumbent users, respecting international agreements with neighboring countries, and increasing the quality of service for new users. A good way of implementing a suite of policies is to grade them by importance and use a sequence of optimizations, where each optimization protects the results of previous optimizations. After each optimization, the objective function is turned into a constraint on the feasible solutions for all subsequent optimizations. In this way, subsequent optimizations become tiebreakers for solutions to the previous optimization problems. Below we give examples of different policies considered for the BIA.

With a band plan chosen, it is possible that some TV stations will need to be placed in the mobile band. As the mobile licenses are to be auctioned and their usability depends on the level of interference from TV stations, it is important to decide on a notion of quality for a mobile license. As the new licenses are to be sold, the regulator wants to maximize the quality of the licenses according to the defined notion of quality. We will return to this later.

The FCC has chosen the following order of importance for their policies:[21]

1) Protection of incumbents
2) Border considerations with Mexico and Canada
3) Information from broadcasters
4) Quality of mobile licenses
5) Nationwide impairment thresholds
6) Relocation of TV stations

This order of optimizations shows that the top priority for the FCC is to protect TV stations from interference.

1. Protection of Incumbents

The Spectrum Act[22] passed by Congress allowed the use of spectrum optimization in the BIA under the condition that "all reasonable efforts" were taken to preserve the coverage areas and served populations of TV stations.[23] This has become the most important policy requirement in implementing the BIA. The policy is so important it is not included as an objective function, where one could protect as many TV stations as possible, but rather, it is included as hard constraints in all optimizations to ensure that *all* TV stations are protected. Consequently, all optimizations have a collection of interference constraints that are designed to protect TV stations. We discuss these in detail in Section II(C)(1).

2. Border Considerations

The success of repacking will be highly dependent on the agreements a regulator has with neighboring countries. A single country can decide to repack parts of its spectrum for repurposing, but if there is no coordination with neighboring countries to also do some amount of spectrum repacking, then the quality of service that a regulator can provide for new services is limited. This is particularly important in a European setting where each country has several neighbors to consider and countries are smaller than in North America. For the BIA, the FCC has an agreement with the Canadian regulator to repack all Canadian TV stations as part of the repacking process.[24] Because there are a smaller number of TV

21. *Competitive Bidding Procedures for Broadcast Incentive Auction 1000*, GN Dkt. No. 12-268, Public Notice, 29 FCC Rcd. 15,750, Appendix C, F (2014).
22. Middle Class Tax Relief and Job Creation Act of 2012*,* 26 U.S.C. § 1 (2012).
23. Spectrum Act, *supra* note 12, at § 6403(b)(2).
24. FCC, STATEMENT OF INTENT BETWEEN THE FEDERAL COMMUNICATIONS COMMISSION OF THE UNITED STATES OF AMERICA AND THE DEPARTMENT OF INDUSTRY OF CANADA RELATED TO THE RECONFIGURATION OF SPECTRUM USE IN THE UHF BAND FOR OVER-THE-AIR TELEVISION BROADCASTING AND MOBILE BROADBAND SERVICES (2015),

stations in Canada than in the US, Canada doesn't need to hold a reverse auction for relinquishment of spectrum usage rights in order to be able assign Canadian TV stations channels within the repacked TV band.[25] Seen from the perspective of the FCC, having the ability to repack the Canadian TV stations is critical, as 75% of the Canadian population lives within 100 miles of the U.S. border. If the Canadian TV stations covering this area were fixed to their current channels, they would create a lot of inflexibility in the optimizations and those TV stations on channels in the new mobile band would cause impairments to the U.S. mobile licenses close to the border.

To understand the possible consequences, consider the southern border with Mexico. Here, the international agreement is different from that with Canada. Mexico has agreed to reassign all its TV stations to channels below channel thirty-seven. The Mexican TV stations must be protected on these new channels from interference from U.S. TV stations.[26] If the FCC adopts a large clearing target, then the highest channel in the TV band will be below 37 and Mexican TV stations will impair many of the new U.S. mobile licenses near the border.

Los Angeles is the second most populated license area in the United States. For the initial 126 MHz clearing target, which allows for up to ten mobile licenses in each area, there are only five impairment-free licenses in Los Angeles and only four in San Diego.[27] Part of the explanation is that there are a lot of TV stations in Los Angeles and it is therefore difficult to find channels for all of them, but the fixed Mexican stations operating on channels thirty to thirty-seven will cause impairments to the first six mobile licenses. Since the initial bidding price (the reserve price) for a mobile license in Los Angeles is $100,000,000, the loss in value is presumably quite high. In contrast, the value of being able to repack TV stations in neighboring countries can be seen in Seattle, where TV stations covering Vancouver could impair the mobile licenses. In fact, all ten licenses in Seattle are completely impairment free.[28] If the Vancouver based TV stations could not be repacked, the four TV stations in the Vancouver area currently broadcasting on channels above twenty-nine would cause impairments to U.S. mobile licenses.

https://transition.fcc.gov/ib/sand/agree/files/PASIIC.pdf [https://perma.cc/8AXC-TBGB].

25. *Id.*

26. FCC, LETTER TO RICARDO CASTAÑEDA ALVAREZ (2015).

27. FCC, APPENDIX A: FORWARD AUCTION BLOCKS IN EACH PEA (2015).

28. *Id.*

3. Information from Broadcasters

Generally, there are three types of spectrum repacking:

1) Incumbents are fixed and not repacked.
2) Incumbents are repacked within their assigned band.
3) Incumbents are moved to another band at the cost of decreased coverage, but receive monetary compensation.

With the use of optimization techniques, a regulator can implement all three scenarios and see which will generate the required amount of new spectrum. Optimization techniques are ideal for experimenting and answering "what if" questions like the three above. The FCC decided to go with the third type of repacking where information from the TV broadcasters is needed to identify which other bands they are willing to accept and for what monetary reward.

Consider the schematic representation of the TV bands in Figure 6.

FIGURE 6

Off Air	Low-VHF	High-VHF	UHF

Prior to the reverse auction, each TV station submits which relinquishment options it will likely consider in the reverse auction. Prior to the first stage of the reverse auction the repacking of the TV bands will move as many stations as possible as far to the left as possible in the above diagram. In each round of the reverse auction, a TV station is offered a compensation amount for each of its relinquishment options. If a station decides that it does not want to be in the band it is currently in for the price offered, it can choose to move to the right in the above diagram, i.e. move up in frequency band, but with the cost of losing compensation as the higher bands/options have lower price offerings attached. A TV station can never move left during bidding. Using these rules, the repacking is used to facilitate the reverse auction.

The overall goal is to clear the upper part of the UHF band, and thus UHF stations are the stations that are most important to move away from their pre-auction bands. Having fewer stations in the UHF band will decrease the potential impairments to the mobile licenses and increase their quality and hence their value.

4. Quality of Mobile Licenses

The ideas presented in this section apply in general to determine the quality of spectrum shared by different types of services. We use the BIA with TV stations and mobile users as an example.

The repacking process in the BIA is designed with the flexibility to account for the case where it is not possible to repack all TV stations in the new TV band. While this was not the case in the final band plan that was adopted at the end of stage 4 of the BIA (there were no impairments in the final 84 MHz band plan), the process was designed to allow for the possibility of some stations broadcasting on channels in the new mobile band. In addition, TV stations in neighboring countries can cause interference to domestic mobile licenses. It is therefore important to be able to estimate the potential inter-service interference between the mobile network and the TV broadcasters. In Section II (C)(2), we will discuss the details of how the FCC decided to estimate the inter-service interference. For the moment, let us assume that we have a way of determining the level of impairment caused to a mobile license by a TV station broadcasting on a channel in the new mobile band. Such impairment will usually be specified as a percentage of the population living in the area covered by the license.

The value that a mobile operator will assign to a license depends on many factors, including the population of the area covered by the license. This can be impacted by impairments caused by TV stations being assigned adjacent to or in the new mobile band.

In a clock auction for spectrum where different blocks of spectrum within a geographic area are largely substitutable, the auctions are often comprised of two phases. In the first phase, bidders bid for generic blocks of spectrum (the allocation phase), and in the second phase, winners from the first phase are allowed to bid to express preferences for specific frequencies (the assignment phase). The BIA forward auction is no exception. When the closing conditions for the auction are met, the mobile operators that won generic lots in the first phase will bid for specific frequency blocks. The value assigned to specific licenses by the mobile operators is different because the licenses can be impaired differently by interference. Thus, it is important for the mobile operators to be able to distinguish licenses when bidding. Ideally, spectrum auctions only have "clean" spectrum where bidders do not have to take impairments from other services into account. However, in the case of the BIA, instead of having one type of generic licenses, the FCC divides all spectrum blocks into three categories to take into account varying ranges of impairment. Category 1 contains blocks impaired up to 10%, Category 2 consists of blocks impaired between 10% and 50%, whereas Category 3 blocks are impaired more than 50% and are not for sale. Mobile operators will prefer unimpaired spectrum, so maximizing the number of Category 1 blocks is preferred.

The decision not to sell spectrum blocks that are more than 50% impaired (Category 3) has several benefits and is a way to mitigate the problem of conservative interference constraints preventing economic value to be unlocked. A block with such a high impairment will be of limited use for mobile operators. The decision not to sell Category 3 blocks can help reduce impairments to other licenses too. The optimization is designed to take advantage of these blocks and collect as many stations on channels interfering with them. While this will drive up the impairment for the Category 3 spectrum block, which is not being auctioned anyway, it does so to potentially lower the impairment to other auctioned blocks.

The total impaired population gives a global measure for the quality of a station-to-channel assignment. This will reflect how many TV stations are in the new mobile band, how much interference they cause, and the population in the area of the mobile license. Keeping the total impaired population low will create a solution, which is overall of higher quality, and thus more likely to generate higher proceeds when mobile licenses are sold, although there may be local areas where the impairment is high and the quality of the license consequently lower. By using this metric, the licenses covering densely populated areas, where high quality spectrum is in most demand, are favored over licenses covering rural areas.

The international agreement between the FCC and the Canadian regulator includes the condition that the FCC cannot minimize the impairment caused to U.S. mobile licenses at the cost of impairing Canadian mobile licenses. The agreement means both countries must be treated equally in the optimization. Spectrum optimization does not need the international agreements with neighbors to be of the form between the United States and Canada. It can allow for any number or type of agreements between neighboring countries. As in the case of the BIA, there are different agreements with Canada and Mexico, and both are incorporated into the optimization.

5. Nationwide Impairment Thresholds

The FCC has a sequence of four optimizations to maximize the quality of the new mobile licenses.[29] The first of these seeks to

29. FCC, CLEARING TARGET OPTIMIZATION, Appendix C, § 2, fig. 1 (2015), https://wireless.fcc.gov/auctions/incentive-auctions/DA-15-1183_Appendix_C_update.pdf [https://perma.cc/JDC3-PH6P].

curtail the maximum impairment percentage of both the United States and Canada. From a computational perspective this optimization is difficult and time-consuming (for a problem of the size considered in the BIA it could take a week for the 126 MHz clearing target). We will go into the details of this in Section II(D). The objective function is continuous and finding the true minimum of this function and a station-to-channel assignment that provides this minimum is extremely difficult. Unless the national impairment percentages are zero, it is quite likely that two different station-to-channel assignments will have different national impairment percentages. Thus, there will be a very small set of station-to-channel assignments that will provide the minimum of the first of the quality optimizations. Each of these station-to-channel assignments will be very difficult to find and it may be pointless to include further optimizations to break ties between solutions. On the other hand, the subsequent optimizations fulfill different spectrum management policies—so a regulator might be interested in using these. A way around this problem is to specify a lower bound for the national impairment percentages. The regulator can specify a minimum threshold of national impairment it is willing to accept. This means that the first optimization will stop when any station-to-channel assignment is found with country-specific impairment percentages at the threshold.

The threshold approach strikes a balance between lowering the impairment percentage and the other objective functions to allow for maintaining good assignments with respect to multiple objectives. The threshold implies that the new TV band will not get too tightly packed and thus gives some maneuverability and "wiggle room" in the TV band. The reason for this is found in the design of the reverse auction.

In the reverse auction, a TV station is only asked to bid if the station is accommodated in its pre-auction band given the current set of stations that need assigned channels in that band. Firstly, if the TV band is packed very tightly then there might not be any room for the considered TV station in the TV band at all. The station would not be asked to bid at all and its compensation would be its initial price. Secondly, as mentioned previously, finding feasible station-to-channel assignments is a so-called NP-complete problem. Thus in some instances it can be very difficult to find a feasible station-to-channel assignment and difficult to prove that a station can be accommodated in its pre-auction band. These "feasibility checks" are done several thousand times between each round of the reverse auction. When a bid is processed, the potential compensation to that TV station is lowered. Given the complexity of some of these problems, the FCC set a time limit for finding a feasibility solution.[30]

Moreover, the density of the TV band will vary geographically, with the densest areas being the metropolitan high-value areas. If the TV band is tightly packed at the beginning of the reverse auction, it is possible that feasibility checks for many stations in the metropolitan areas timeout because the channel-assignment problem is too difficult to solve in the allowed time. When a feasibility check times out, it is not possible to guarantee that the station can get a channel in its pre-auction band, and thus its bid cannot be processed. In other words, timeouts are treated like the feasibility check returning the answer that it is not possible to move the station to its pre-auction band. Recall that due to the rules of the reverse auction, stations can only move up in frequency band. If a station is placed in the UHF band, it is out of the auction and will never move away from the UHF band. Thus, if a feasibility check for a UHF station shows that it is not possible to accommodate the station in the new UHF band with the current occupancy, it is not possible in the future as only more stations will be added. Consequently, the station is fixed in its current band and is locked out of bidding in the current stage of the auction. If the stage is the final stage, then the station will be a winner and will be given the compensation presented when it was asked to stop bidding. If the timeout happens to be a false negative, then it can be quite costly for the FCC. Allowing for "wiggle room" in the TV band makes the feasibility checks easier and thus potentially allows more bids to be processed and lowers the total compensation to all TV stations. The compensation comes from the proceeds of the forward auction and the auction does not end until the proceeds meet at least the compensation. Including the national impairment threshold provides a buffer that potentially could minimize the number of stages the BIA has to go through.

6. Relocation of TV Stations

The transition of TV stations to new channels can be a time-consuming task. Mobile licenses will not be ready for use before the occupying TV stations clear the spectrum by moving to their new channels. The transition will be easier if few stations need to move channels. Relocating TV stations to different channels is a complicated procedure that usually involves installing new broadcast equipment or replacing antennas. The antennas are usually located on high buildings or towers, and some towers are in

30. *Incentive Auction Task Force Releases Information Related to Repacking; Announces Workshop/Webinar to Provide Additional Detail,* GN Dkt. No, 12-268, Public Notice, n.9 (released Jan. 9, 2014).

locations that are only reachable by helicopter. Furthermore, there are a limited number of experienced tower crews available to perform these transitions. It would therefore help the transition process along if many of the TV stations were assigned to their original channel.

It is easy to imagine that the relocation of TV stations is a costly procedure, but also that it is cheaper for some stations than other stations. For example, a TV station broadcasting where the antenna is located in an easily accessible place is much cheaper to relocate to another channel than a TV station broadcasting from an old TV tower at the top of a mountain. In that case, the equipment on top of a mountain requires the construction of a completely new tower because the old tower cannot handle the extra weight of the new equipment. In the case of the BIA, relocation costs will be paid from auction proceeds; it is in the interest of the FCC to minimize these costs.

While the relocation costs can be large, they will not take priority over the overall goals of repacking the spectrum and increasing the quality of the mobile licenses. The relocation cost considerations are implemented as the last of the spectrum management policies. The optimizations do not run until the end of the auction when the band of each TV station is known.

The general objective functions to consider for the relocation of incumbents to new frequencies are:

- Maximize the number of incumbents that remain on their current channel.
- Minimize the relocation costs.
- Minimize the aggregate amount of small interference between incumbents.
- Minimize the use of the extreme frequencies in the band to avoid spillover effects to neighboring bands.

To implement several of these functions a regulator needs to rank the objectives and implement them sequentially such that subsequent objective functions become tiebreaker constraints among optimal feasible solutions to the previous objective function.

The third above-mentioned objective function is related to the way interference between TV stations is modeled. In Section II(C)(1) we will discuss the TV-to-TV interference constraints and it will become apparent that there is a threshold for interference between two TV stations. If the interference predicted between TV stations is smaller than the threshold, then it is incorporated in the third optimization above, which tries to minimize this interference.

C. Constraints

Constructing a station-to-channel assignment fulfilling all the goals of the spectrum management policies described in Section II(B) is done using mathematical optimization models. The spectrum policies are incorporated as objective functions and constraints in these models. The constraints are of two kinds, defined by the interference TV stations cause to each other or by the inter-service interference between TV stations and the mobile network. In this section, we will describe both of these constraints.

1. TV-to-TV Interference Constraints

The main types of constraints driving the repacking process are the constraints modeling interference between TV stations. The main driver behind the design choices taken by the FCC was the mandate given by Congress in the Spectrum Act, which allowed the BIA and repacking of the TV band under the condition that all reasonable efforts were taken to preserve the coverage area and populations served by TV stations, i.e., protect the incumbents.[31] The implemented constraints are based on interpretations of the keywords *coverage area* and *population served*.

The *coverage area* of a TV station is determined by the choice of radio wave propagation model. In Section II(E), we discuss propagation models in more detail, but for now, let us assume that a propagation model can, at a specified geographical location, predict the signal strength of a TV station broadcasting on a particular channel. The FCC has divided the whole of the United States into 2x2 km cells. The Longley–Rice propagation model is used to determine the field strength at the population centroids of the 2x2 km cells for each TV station on each possible channel. The propagation characteristics depend on frequency, and thus the field strengths will be different for different channels. For a TV station on a given channel, the coverage area is defined as the area covered by the cells where the signal strength exceeds a certain decibel threshold set forth by the FCC for a percentage of the time. This will generally give rise to a contour within which cells can receive the signal. When these calculations are done for all TV stations on all channels, we can determine the cells where the signal between two TV stations interfere.

The keyword *population served* can be interpreted in different ways. The most obvious is that the exact same cells should receive a TV station after the repacking. This is problematic because the coverage area depends on the broadcast channel. Preserving this definition of *population served* would be difficult. The

31. Spectrum Act, *supra* note 12, at § 6403(b)(2).

interpretation adopted by the FCC is to use the total population served on the day the Spectrum Act was signed into law by Congress in 2012, and consider the reduction caused by any individual station-channel pair to the population served.[32] If the reduction is 0.5% or more, then a constraint is included that prevents the two stations from operating on the specific channels at the same time.[33] In this way the interference constraints become pairwise constraints between two TV stations *s* and *s'* broadcasting on channels *c* and *c'*, respectively, and mathematically they are very simply defined by:

$$x_{s,c} + x_{x',c'} \leq 1$$

where $x_{s,c}$ is a binary variable that is 1 if station s broadcasts on channel c. The constraint explicitly prevents stations s and station s' to operate on channel c and c', respectively, at the same time.

In addition to the pairwise TV-to-TV constraints, the repacking procedure is subject to a domain constraint, which ensures that all stations must be assigned to some channel, including off-air if that is one of the relinquishment options selected by the station. This is expressed mathematically as:

$$\sum_{c \in Domain(s)} x_{s,c} = 1$$

where the domain of a station consists of the channels the station can possibly be assigned to. This depends on the pre-auction band of the station along with its chosen relinquishment options. Furthermore, the Mexican stations do not repack in the BIA so any pairwise constraints between a Mexican TV station and a U.S. TV station are resolved by removing the problematic channels from the domain of the U.S. TV station.

During repacking, these pairwise TV-to-TV constraints are the main constraints. The repacking process is part of the feasibility check done thousands of times in a short span of time between each round of the reverse auction; therefore, the constraints need to be as simple as possible. These pairwise constraints strike a balance between simplicity and strict interpretation of the rules.

One of the shortcomings of the pairwise constraints is that a TV station may go from not having any interference from neighboring TV stations to experiencing interference from all neighbors, which is totally more than 0.5% but each individual interference is less than the 0.5% threshold. To acknowledge this, the FCC implemented an objective function in the relocation

32. BIA Report and Order, *supra* note 17, at n.19.
33. *Id.*

package of optimizations that seeks to minimize the aggregate interference experienced by each station.[34]

Another way to acknowledge the simplifications of the TV-to-TV interference constraints is to include a cap on the aggregate population loss. The FCC chose not to implement this strategy because a TV station will generally be moved to a lower channel and therefore broadcasts on a lower frequency with better propagation properties. The overall effect is an excepted increase in the coverage area of the given TV station.

The conventional use of propagation models to predict interference (and the one chosen by the FCC presented above) is determining if a grid point is either interfered or not. However, as we will discuss in Section II(E), it is not that simple. Propagation models are stochastic and there is a distribution of ratios of desired and undesired signal field strengths at each grid point. Using the ratio distribution rather than the binary decision—for determining interference constraints or an interference objective function—would give a better use of the spectrum. However, the benefit of the simple pairwise constraints are clear when optimizations are conducted repeatedly over a short span of time with the current level of computational power available to the FCC.

2. Inter-Service Interference ("ISIX") Constraints

An essential part of the repacking procedure is to allow for some minimal number of TV stations to be assigned in the mobile band. Therefore, it is important to model the inter-service interference in the right way to make best use of the spectrum. As with regular interference between TV stations discussed above, if the interference models are too conservative, then the spectrum is not fully utilized, and on the other hand if the interference predicted is too low, then the quality of mobile licenses is compromised.

34. *Broadcast Incentive Auction Scheduled to Begin on March 29, 2016, Procedures for Competitive Bidding in Auction 1000, Including Initial Clearing Target Determination, Qualifying to Bid, and Bidding in Auctions 1001 (Reverse) and 1002 (Forward)*, GN Dkt. No. 12-268, Public Notice, 15 FCC Rcd. 78, n.273 (Aug. 11, 2015) [hereinafter *Auction 1000 Bidding Procedures*].

FIGURE 7

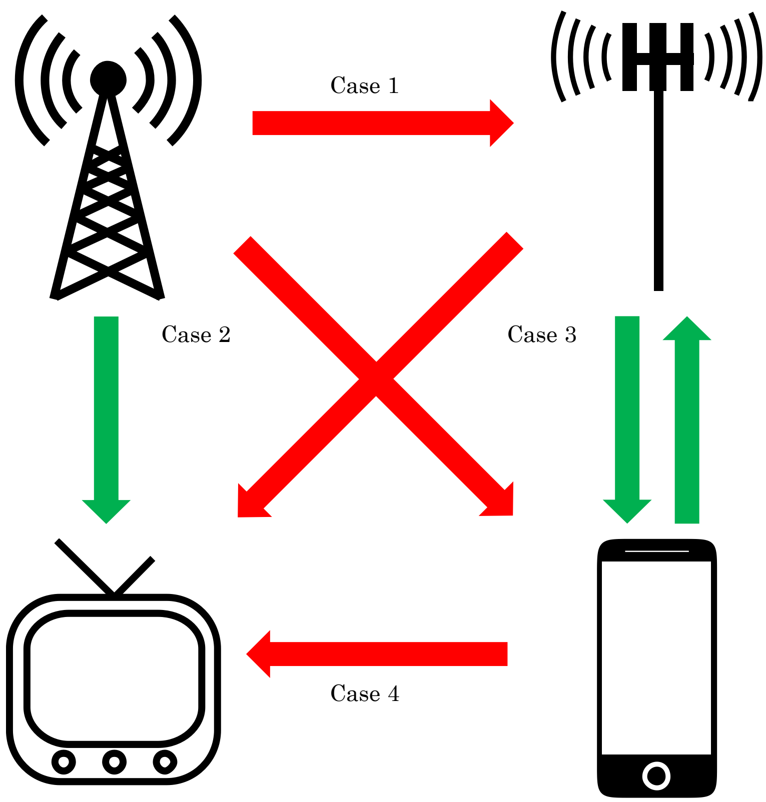

Figure 7: Inter-service interference between mobile networks and TV broadcasters

Inter-service interference between TV broadcasting and mobile networks arises when there is interference caused by TV stations to mobile networks, and interference caused by the mobile networks to the signal from TV stations. A typical mobile license[35] consists of two parts of spectrum, an uplink part where handheld devices transmit signals to base stations, and a downlink part where base stations broadcast signals to be picked up by handheld devices. Since interference can appear in uplink and downlink there are essentially four different cases to study:

35. There are two main approaches to channel configuration for mobile communication, Frequency Division Duplex (FDD) and Time Division Duplex (TDD). FDD requires two separate channels with a guard band between for receiving and sending information. In TDD, information is sent and received on the same channel, but it requires very accurate timing of the equipment to separate the transmissions.

- **Case 1**. The TV signal from a TV station can interfere with the signal from handheld devices such that the receivers on the mobile base station cannot distinguish the signals.
- **Case 2**. The signal from a TV station can interfere with the transmitted signal from a mobile base station such that the receivers in a handheld device cannot distinguish the signals.
- **Case 3**. The transmitted signal from a mobile base station can interfere with the broadcast TV signal such that the TV receivers cannot distinguish the signals.
- **Case 4**. The transmitted signal from a handheld device can interfere with the broadcast TV signal such that the TV receivers cannot distinguish the signals.

A regulator must decide how to model each of these four types of interference. In the first three cases, the signals causing interference come from antennas mounted on high buildings or towers. As these environments are generally similar (signals transmitted from high sites), it is reasonable that the same propagation model is used for these cases. However, broadcasting equipment is different from mobile transmitters and it operates at much higher power levels, so different parameters must be used in the propagation models to study the interference potential. In Case 4, the transmission is coming from a mobile device, so the antenna height is usually 1.5 meters above ground and transmission power is usually low, which is quite different from the other three cases. Furthermore, the signal from a handheld device is transmitted from below rooftop, where it is difficult to get good predictions of signal strength. Therefore, Case 4 interference must be handled differently than the other three cases.

There are many choices in regard to modeling interference and the examples given above are just some of what a regulator could choose to do. It may be reasonable to include the mobility of handsets in the interference modeling. The mobility means that Case 2 and Case 4 sometimes are given less importance, and coordination between operators is done to avoid Case 1 and Case 3 interference.

The remainder of this Section discusses how the FCC has implemented the inter-service interference (ISIX) in the BIA. This will serve as a particular illustration of the more general requirements discussed. Based on the Spectrum Act and its clause regarding the protection of TV stations, the FCC has adopted the overall philosophy to allow no interference from mobile technology to TV signals.[36] This was implemented as a zero percent

36. *Expanding the Economic and Innovation Opportunities of Spectrum Through*

interference threshold for Case 3 and Case 4 interference despite predictions being probabilistic and that interference may occur, although with very low probability.

The FCC uses the Longley-Rice Propagation Model to model the interference Cases 1, 2, and 3.[37] We will discuss this in more detail in Section II(E). Case 4 interference is modeled by enforcing a fixed separation distance between the TV station contour (the edge of the coverage area determined by the Longley-Rice model) and handheld devices. The reason for this different approach is that the signal emitted by handheld devices is not expected to be able to cause interference to signals from TV stations in a distance of more than a few kilometers. The fixed separation distance implemented is therefore 5 km for co-channel and 0.5 km for adjacent channel operations. In fact, to avoid any doubt, all four cases of inter-service interference could be modeled by enforcing a fixed separation distance. However, the fixed separation distance is a conservative way to model interference, as it does not include any information about technical characteristics of transmitting devices, terrain, or population density. Enforcing this approach for all cases of interference would be a very inefficient use of the spectrum.

A general problem faced when modeling inter-service interference between incumbents and new users is that the incumbents have existing infrastructure whereas new users have yet to deploy, or even to have designed their system. In the particular case of inter-service interference between TV broadcasting and mobile networks, the location of the TV antennae are fixed and their heights, transmission power, and transmission patterns are known, but the mobile network has yet to be deployed. Under FCC rules, the winner of a license can decide how to build its network to cover the license area under the constraints that limit the service area so that the users will not cause interference to co-channel or adjacent channel TV stations.[38] The FCC requires that the mobile licensees evaluate the interference caused by base stations to TV receivers in the TV contours when the network is established and every time it evolves in a way that could cause extra interference, e.g. a higher antenna, higher power, etc.[39]

To add to the complexity, a technical difficulty with the choice of block sizes has influenced the way the FCC has implemented

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Incentive Auctions, GN Dkt. No. 12-268, Second Report and Order and Further Notice of Proposed Rulemaking, 29 FCC Rcd. 13,071, 13,109–10 (2014) [hereinafter Second Report and Order].

37. *See Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, GN Dkt. No. 12-268, Third Report and Order and First Order on Reconsideration, 30 FCC Rcd 12,049, Appendix D, Sections III, IV (2015) [hereinafter Third Report and Order].

38. *Id.* at para. 32.

39. *Id.* at paras. 58–75.

inter-service interference. Digital TV stations have channels that are 6 MHz wide and mobile licenses are usually 5 MHz wide.[40] This means that the interference is not only determined by geography and technical equipment, but is also highly dependent on the amount of spectral overlap between the channel and the mobile license.

Let us consider the Case 1 and Case 2 interference caused by TV stations to mobile licenses. To model these cases, the TV station is fictitiously placed on channel thirty-eight and the propagation model will estimate the field strength at each of the 2x2 km grid points. The field strength is then compared to the appropriate interference field strength threshold for each spectral overlap. These thresholds are determined independently for Case 1 and Case 2 and the more interference allowed, the lower the spectral overlap. Generally, the FCC has allowed a higher threshold for Case 2 than for Case 1 as handheld devices usually move around.[41] The Case 1 and Case 2 interference caused by each TV station on channel thirty-eight to mobile licenses are then extrapolated to all other channels in the mobile band.

Modeling Case 3 interference caused by mobile base stations to TV receivers is done in a similar fashion as for Cases 1 and 2. Here, a TV station is fictitiously placed on channel thirty-eight and its coverage area is calculated using the Longley-Rice Propagation Model. The idea is to compare the field strengths of the TV signal and signal from a mobile base station at each 2x2 km grid point in the coverage area. If the ratio between the wanted signal and the unwanted signal is below a certain threshold (depending on the spectral overlap) the grid point is considered impaired.[42] These interference ratios are calculated for all potential locations of mobile base stations. The potential locations are defined to be the grid points of a 10x10 km grid across the United States. If a mobile base station at one of the grid points impairs a single 2x2 km grid point in the coverage area of a TV station, then the whole county in which the 10x10 km grid point is located in is considered impaired for the mobile license.[43] These Case 3 impairments are quite severe for mobile licenses, and choosing this 0% threshold is a rather conservative policy that limits the use of the mobile licenses. We believe this will be of particular importance in urban areas where the mobile licenses are in high demand.

As the goal of the inter-service interference modeling is to determine the quality of a mobile license in the presence of a TV station operating in the mobile band, such quality is expressed as a

40. *Id.* at Appendix C, Section IV(F).
41. Second Report and Order, *supra* note 36, at paras. 11–12.
42. *Id.* at para. 8.
43. *Id.* at para. 18.

percentage of the population in the mobile area that are considered impaired in the sense of the Case 1 to Case 4 described above. Given a specific station-to-channel assignment, we can for each mobile license collect all the 2x2 km grid points in the license area that are impaired in either the up- or downlink part of the license by TV stations assigned to the new mobile band.

a. Mathematical Construction of ISIX Constraints

There are roughly 2.5 million of the 2x2 km grid points covering the entire United States. Searching all of these points to determine impairments for each mobile license in each station-to-channel assignment is a computationally expensive operation. The fine-grained 2x2 km data are therefore impractical for implementation in the optimization model. The solution adopted by the FCC is to consider impairments on a county-level instead. There are around 3,100 counties in the U.S., which from a computational perspective is much more manageable than the 2.5 million grid points.[44] For a given mobile license, if more than 10% of the population in a county is impaired by the presence of a TV station on a channel in the mobile band overlapping with the mobile license the county is considered impaired for the mobile license.[45] The license area is divided into several counties.[46] The impairment percentage of a mobile license is determined by the percentage of the population living in impaired counties for that mobile license. Using county-level impairments approximates the impairments given by the 2x2 km grid points. Once a final station-to-channel assignment is determined, then the 2x2 km data will be used to find the "correct" impairments to each mobile license. These impairments will determine the quality of the mobile license in the forward auction.

With the county-level impairments, the actual inter-service interference (ISIX) constraints implemented by the FCC are given by the following examples:

$$x_{s,c} \leq y_{a,l}$$

44. *USGS FAQs,* U.S. GEOLOGICAL SURVEY, https://www2.usgs.gov/faq/categories/9799/2971 [https://perma.cc/N9A9-L5CB] (last visited Mar. 13, 2017).

45. *Auction 1000 Bidding Procedures*, *supra* note 34, at paras. 20–21.

46. Each license area is one of the so-called Partial Economic Areas (PEA). The U.S. is divided into 416 of these PEAs. This was established in the Incentive Auction Report and Order. *See Wireless Telecommunications Bureau Provides Details About Partial Economic Areas*, GN Dkt. No. 12-268, Public Notice (released June 2, 2014), https://apps.fcc.gov/edocs_public/attachmatch/DA-14-759A1_Rcd.pdf [https://perma.cc/F27F-2TD9].

$$\sum_{a \in A_l} pct_{a,l}\, y_{a,l} = \rho_l$$

Here the decision variables $x_{s,c}$ are the binary variables from the TV-to-TV constraints that are 1 if station s is assigned channel c. The variable $y_{a,l}$ is also a binary variable that is 1 if county a is impaired for mobile license l. The first ISIX constraint is included if the propagation modeling for Cases 1–4 determines that station s on channel c impairs county a for mobile license l. The variable l is the impairment percentage of mobile license l. The coefficient $pct_{a,l}$ is the percentage of the population in the area of license l that live in county a. The second ISIX constraint is that the impairment percentage of a mobile license is the sum of the percentages from impaired counties. The set A_l is the set of counties making up the area of license l.

Recall that the first of the optimizations to maximize the quality of the mobile licenses was to minimize the nation-wide impairment percentage for both the United States and Canada. The nation-wide impairment percentage is determined by the impairment percentages of each license and can thus be calculated as:

$$\frac{\sum_{l \in L_k} w_l \rho_l}{\sum_{l \in L_k} w_l}$$

where w_l is the population of the license area l and L_k is the set of mobile licenses in country k. There are two nation-wide impairments, one for the United States and one for Canada. The first of the quality optimization is to minimize the maximum of these two figures.

Recall that if a mobile license is more than 50% impaired then it is not for sale.[47] Therefore, if there is a set of TV stations impairing a mobile license to more than 50%, then in terms of the quality of the mobile license in the forward auction it would not matter whether the license is 50%, 70%, or 100% impaired. Including this distinction would allow for more stations in the mobile band at a limited cost because the licenses that will be affected are already impaired by other TV stations. As we said in Section II(B)(4), this gives more flexibility in the repacking process.

The explicit implementation of the 50% impairment cut-off can be done using an extra decision variable N_l which is 1 if license l is more than 50% impaired and zero otherwise.

$$\rho_l \leq 0.5\, N_l + 0.5$$

47. *Auction 1000 Bidding Procedures*, *supra* note 34, at para. 24.

$$\rho_l \geq N_l$$

If the impairment percentage l is more than 0.5 (or 50%), then for the first inequality to be valid the variable N_l must be 1. If N_l is 1, then the second inequality implies that l also must be 1. Together these two constraints model the fact that the sell value of the mobile license is 0 whether the impairment percentage is 50.1% or 100%, so you may as well set the percentage to be 100%.

D. Computationally Effectiveness

The BIA not only broke new ground in terms of novelty in spectrum management, but also in terms of the sheer size of the optimization problems that lie on the verge of what is computationally feasible. This section describes a bespoke optimization tool developed for processing bids in the reverse auction. It will furthermore discuss the simplifications necessary to enable a solution to the inter-service interference optimization problems to be found and propose a tool to enable efficient spectrum sharing in a more general situation.

1. SATFC: TV-to-TV Interference Optimizations

In each round of the reverse auction, the TV broadcasters are presented with a range of relinquishment options as an alternative to their currently held option. When the auction stops, all TV stations must be able to be assigned a channel in the band corresponding to their currently held option.[48] The assignment found must satisfy the pairwise TV-to-TV interference constraints discussed in Section II(C)(1).[49] To maintain feasibility, creating new assignments is a crucial component of processing the relinquishment options submitted by the TV stations. Being able to find a new assignment determines whether a station can remain participating in the auction or not. As the reverse auction progresses, more and more stations will choose to drop out as they reject the prices presented to them for any of their relinquishment options. The bands will slowly fill up with stations that have dropped out. If a TV station cannot be given a channel in its pre-auction band, then its price for the current option is frozen and it will not be able to bid anymore in this stage.[50] If the auction ends after that stage, then a frozen station is a winner in the auction.[51] In addition to the pre-auction band checks, relinquishment option checks must be carried out to check which option(s) of a TV station

48. Third Report and Order, *supra* note 37, at Appendix D, § 5.2.
49. *Id.* at Appendix D, § 2.5.
50. *Id.* at Appendix D, § 5.3.
51. *Id.* at Appendix D, § 1.3.

can be satisfied.[52] Combined, these checks are carried out several thousand times between each round of the reverse auction.

The optimizations prior to a stage of the reverse auction[53] produce, among other things, the impairments, i.e. the set of stations that would share the new mobile band with mobile operators. These stations are not participating in the reverse auction as they are already assigned to their pre-auction band. These sets of stations and channels are fixed in the checks between rounds of the reverse auction.[54] All other stations are assigned to channels at or below the highest channel given by the clearing target (channel 29 for clearing target 126 MHz).[55] Therefore, all the difficult sharing considerations between TV stations and mobile devices can be ignored for non-impairing stations and we are left with just the pairwise TV-to-TV constraints, and thus a graph coloring problem. We mention that the graph coloring problem is NP-complete, which means that there is no efficient algorithm to obtain a graph coloring. The difficulty of the problem depends on the complexity of the graph and the number of colors allowed. In the case of the BIA, the graph is the one in Figure 4 with more than 2,900 nodes and the clearing target limits the number of colors. This poses a potential problem for the reverse auction as a new graph coloring must be found several thousand times in each round, and the coloring will have to come fast in order not to delay the next round of the auction.

The FCC developed SATFC—a bespoke piece of graph coloring software designed specifically for the inter-round checks—to determine whether a station could continue bidding in the reverse auction or should be frozen.[56] The purpose of SATFC is to quickly find a feasible solution to a given problem, if it exists. Furthermore, SATFC must be accurate in its answers. False positives (identifying that there is a feasible channel for a broadcaster where there is none) could be fatal for the entire auction and false negatives (failing to recognize there is a feasible channel) are costly. A false positive means that a station does not have its relinquishment price frozen so it continues to bid in subsequent rounds, even though it is not possible to assign it to a channel in its pre-auction band. However, given the auction rules, if this station at some point decides to drop out of the auction, it must be provided a channel in its pre-auction band. However, in the case of a false positive this would not be possible without breaking any interference

52. *Id.* at Appendix D, § 5.3.
53. *See infra* Figure 1 Flow Chart, U.S. Broadcast Incentive Auction.
54. Third Report and Order, *supra* note 37, at Appendix D, § 2.5.
55. *Id.* at Appendix D, § 2.4.
56. *Feasibility Checker*, GITHUB, https://github.com/fcc/SATFC/ [https://perma.cc/27ST-U483] (last visited Apr. 15, 2017).

constraints, i.e. two stations would operate on channels where they caused more than 0.5% interference to each other. If an interference constraint is broken, then the assignment of TV stations to channels do not comply with the requirement of the Spectrum Act about protecting the coverage area of all TV stations.[57] A false negative pre-auction band test means that the TV station has its price frozen earlier (and higher) than is necessary. If the feasibility checking answer were a false negative, the TV station would be stopped from bidding in subsequent rounds as prices decrease, making the cost of clearing spectrum unnecessarily higher.

2. Inter-Service Interference Optimizations

The spectrum optimizations done by the FCC to minimize inter-service interference are extremely expensive in terms of computing power. The main inter-service interference optimization seeks to minimize the maximum amount of interference in the U.S. and Canada simultaneously.[58] The amount of interference is a continuous variable, and mixing the continuous ISIX interference and binary TV-to-TV interference variables will generally give an optimization problem that is difficult to solve.[59] In particular if the objective function is continuous. Add to this the fact that each pair of a mobile license and a county gives a variable.[60] The 126 MHz clearing target produces 10 new licenses in each of 416 areas, which are divided into roughly 3,100 counties. Thus, there are 31,000 variables and as there is a constraint for each station-channel pair impairing a license-county pair, then there are even more constraints. With this setup, the main inter-service interference optimization is on the verge of what is computationally possible, and if the more fine-grained data of 2x2 km grid points were used rather than just approximating using counties, it would require a whopping 25 million variables and many more constraints. The improvements in accuracy of impairments of using the 2x2 km grid points is not likely to be very high. In fact, using the 2x2 km grid points would give a false sense of accuracy as determining whether a 2x2 km grid point is impaired or not is based on signal field strengths coming from a propagation model, which inevitably is just a model of nature, and is therefore not 100% correct at all grid points. The qualities of the mobile licenses are determined by the

57. BIA Report and Order, *supra* note 17, para. 19.

58. *Application Procedures for Broadcast Incentive Auction Scheduled to Begin on March 29, 2016; Technical Formulas for Competitive Bidding*, GN Dkt. No. 12-268, Public Notice, 30 FCC Rcd. 11,034, Appendix C, § 2.4 (2015).

59. In mathematical terms the optimization problem is a Mixed Integer Linear Program as the optimization includes a mix of continuous and integer/binary variables. These are much more difficult to solve than pure Binary Integer Programs (e.g. the graph coloring problem) or pure Linear Programs where there are no binary variables.

60. *See* Section II(C)(2)(a).

interference from TV stations, and this quality assessment is key for the mobile network operators when valuating a license for bidding in the forward auction.[61] To mitigate this, at the end of the inter-service interference optimizations a one-off interference calculation using the higher resolution with the 2x2 km grid points is done to give an accurate picture for the mobile operators to use in the forward auction.

3. SATFC for Dynamic Spectrum Sharing

SATFC is an impressive piece of software using state-of-the-art technology[62] to quickly check whether a new TV station can be added in a particular band containing a set of incumbent TV stations that must remain in the band. SATFC repacks the TV bands to allow more stations and thereby optimize the use of the spectrum. Accommodating a service in a background of incumbent users is the main issue faced in spectrum sharing. In a setting where either both the incumbent services and new service—or just the new services—are automatically retuned to new frequencies, a similar framework to SATFC could be used to facilitate spectrum sharing on a basis that are re-evaluated frequently. The success of such a tool will highly depend on the way in which interference between the users is modeled. In the BIA, the clever modeling of the TV-to-TV constraints as pairwise constraints lead to solving a graph coloring problem and allowing algorithms from this well studied mathematical area to be implemented in the design of SATFC.

E. Propagation Models

The main data generators for the spectrum optimizations are propagation models. As anticipated in Section II(C), propagation models estimate how much interference one service may cause to another. This data can be turned into constraints needed for the optimizations.[63] There are many different propagation models with different features and uses. Some of the more popular models are

61. The impairment of a license is a metric that determines how much of the population in the license area will experience interference from TV stations, and thus mobile network operators can use this to determine how valuable the license is to them.

62. ALEXANDRE FRECHETTE, NEIL NEWMAN, & KEVIN LEYTON-BROWN, SOLVING THE STATION REPACKING PROBLEM (University of British Columbia, 2016).

63. *Auction 1000 Bidding Procedures*, *supra* note 34, at Appendix K.

Longley-Rice[64] (used by the FCC), ITU-R P.452[65], ITU-R P.1546[66], Hata[67] and its extensions. The models are widely used and recognized in the telecommunications industry.

The optimizations framework suggested in this article is not dependent on any one particular propagation model, but will work with any model. The chosen model should be appropriate to modeling the interference in the case at hand. In the BIA example, the FCC used the Longley-Rice model to determine both interference between TV stations and the inter-service interference between TV stations and mobile operators. The FCC could equally well have used the ITU-R P.1546 propagation model for the BIA. In the example of a regulator willing to use optimization techniques to study sharing opportunities between terrestrial services and satellites, the ITU-R P.452 propagation model might be more appropriate to generate constraints protecting the satellite service.

The propagation of radio waves depends highly on the frequency of the waves. At all frequencies, exact simulation in a realistic environment is impossible, and so signal levels must be regarded as random, depending on a range of factors such as atmospheric conditions, the positions and properties of the antennae, the obstacles and scatterers near the antennae and in the whole intervening region.[68]

There are two fundamental questions that any propagation model must answer. First, the question of how to characterize the ensemble of every possible operating conditions and second, how do radio waves propagate under these conditions? There are generally two types of propagation models: path-specific and non-path-specific, also known as point-to-point and point-to-area. Path-specific models (e.g., ITU-R P.452) are mainly used when the locations of both receiver and broadcaster are known and requires detailed knowledge about the terrain in-between the two. An example is fixed terrestrial links that constitute the backbone of a mobile network. Non-path-specific models (e.g., ITU-R P.1546) are used to predict service over a certain area, e.g. for TV broadcasting to determine the area in which the signal can be received. The Longley-Rice model can operate in both path-specific and non-path-

64. A. G. LONGLEY & P. L. RICE, PREDICTION OF TROPOSPHERIC RADIO TRANSMISSION LOSS OVER IRREGULAR TERRAIN – A COMPUTER METHOD (Nat'l Technical Info. Service, 1968).

65. ITU-RADIOCOMMUNICATION SECTOR, PREDICTION PROCEDURE FOR EVALUATION OF INTERFERENCE BETWEEN STATIONS ON THE SURFACE OF THE EARTH AT FREQUENCIES ABOVE 0.1 GHZ (Int'l Telecomm. Union, 2015) [hereinafter PREDICTION PROCEDURE].

66. ITU-RADIOCOMMUNICATION SECTOR, METHOD FOR POINT-TO-AREA PREDICTION FOR TERRESTRIAL SERVICES IN THE FREQUENCY RANGE 30 MHZ TO 3000 MHZ (Int'l Telecomm. Union, 2013).

67. Masayasu Hata, *Empirical Formula for Propagation Loss in Land Mobile Radio Services,* 29 IEEE TRANSACTIONS VEHICULAR TECH 317 (1980).

68. PREDICTION PROCEDURE, *supra* note 65.

specific modes, the latter of which is used for modeling interference by FCC for the BIA.[69]

The random nature of propagation models imply that the output of a model is not a fixed number that expresses the signal strength at a point or over an area, but the model produces distributions of signal strength. The path-specific models give distributions defined at a single point in space, whereas the non-path-specific models define a distribution covering an area. The typical use of a propagation model is to turn the distributions into a deterministic constraint by only considering the percentiles and disregarding the distribution itself. The percentile a regulator chooses depends on the quality of the service it wants to provide, e.g. good service 99% of the time at a particular point, or at 99% of the time for an ensemble of time-location pairs covering a certain area. Specifically, the percentiles are used to determine whether the signal-to-noise ratio is above or below a defined threshold, defining whether the point or area is considered interference free or not.

All propagation models mentioned above are widely used, but are limited to frequencies up to about 6 GHz in the spectrum band. The current challenges faced by the telecommunications industry is to construct and agree on new propagation models at higher frequencies which will be needed for the 5G mobile network. The wavelengths in some of the currently proposed 5G bands are on the millimeter scale.[70] This allows the radio waves to "see" very fine details in building surfaces, whether there are leaves on trees, etc., that radio waves at smaller frequencies are indifferent to. This gives a whole new set of challenges for future 5G propagation models. Many of the current approaches to propagation models for 5G frequencies are fundamentally based on the same techniques used by Longley-Rice and Hata, which are extensive measurement campaigns to gather data on which statistical models can be built. Examples of such models are the 3GPP 3D channel model,[71]

69. OFFICE OF ENG. AND TECH. & FCC, OET BULLETIN: LONGLEY-RICE METHODOLOGY FOR PREDICTING INTER-SERVICE INTERFERENCE TO BROADCAST TELEVISION FROM MOBILE WIRELESS BROADBAND SERVICES IN THE UHF BAND (2015).

70. There is a vast quantity of literature on which bands to use for the 5G network. International discussions focus on several different bands, and different bands will be needed for different technologies. *See, e.g.,* NAT'L INSTRUMENTS, MMWAVE: THE BATTLE OF THE BANDS (2016). Ofcom (the spectrum regulator in the UK) commissioned a suitability study of various 5G bands, which are all mm-bands. OFCOM, 5G CANDIDATE BAND STUDY: STUDY ON THE SUITABILITY OF POTENTIAL CANDIDATE FREQUENCY BANDS ABOVE 6GHZ FOR FUTURE 5G MOBILE BROADBAND SYSTEMS (Quotient Associates, 2015). The ITU and 3GPP have agreed on a plan for research for 5G standards. To align use of mm-wave spectrum on a global scale, the ITU produced a list of viable frequencies between 24 GHz and 86 GHz. ITU-R, PROVISIONAL FINAL ACTS, WORLD RADIOCOMMUNICATION CONGERENCE (WRC-15) 424–26 (2015).

71. Bishwarup Mondal, *TR 36.873, Study on 3D Channel Model for LTE,* TECH-INVITE (Mar. 2017), http://www.tech-invite.com/3m36/tinv-3gpp-36-873.html [https://perma.cc/WEW5-63BQ].

WINNER+,[72] and New York Wireless.[73] An orthogonal and more modern approach is to use ray-tracing software, which needs a very detailed model of the environment including surfaces of buildings, the location of trees and lampposts, etc., to predict how electromagnetic waves propagate. Ray-tracers have been created by both NYU Wireless[74] and University of Bristol Centre for Communication Research[75]. Finally an entirely new approach would be to use numerical simulation techniques to determine propagation of waves in local regions near transmit and receive antennae.

F. Data Requirements

Spectrum optimization manages spectrum more efficiently and allow more users to access spectrum. The optimization problems rely on propagation models as described in Section II(E) to predict interference between users (both inter and intra-service interference). Of course, the quality of the optimizations depends heavily on the quality of the data that are input to the model.

1. Terrain Data

The interference data is the outcome of the chosen propagation model. The propagation model is used to determine the signal strength of a received electromagnetic wave. The input needed differs from propagation model to propagation model. Essential to all propagation models are data about the location of each broadcaster and the equipment used to transmit signals. In particular, the height above ground and the broadcast signal strength are needed. Furthermore, propagation models also need information about terrain. The exact nature of the terrain information differs heavily between propagation models. In the classical Hata Model and its variations, the terrain information is reduced to categorical variables indicating whether the propagation happens in urban, suburban, or in open areas. Of a different type are the Longley-Rice model and the ITU recommended models (P.452 and P.1546) that need detailed terrain information along paths between receiver and transmitter. For the Longley-Rice model, the topography between transmitter and receiver is aggregated to a single parameter expressing the irregularity of the terrain. Several models also require a qualitative expression of the

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72. JUHA MEINILÄ ET AL., CP5-026 WINNER+ FINAL CHANNEL MODELS (2010).

73. MATHEW K. SAMIMI & THEODORE S. RAPPAPORT, STATISTICAL CHANNEL MODEL WITH MULTIFREQUENCY AND ARBITRARY ANTENNA BEAMWIDTH FOR MILLIMETER-WAVE OUTDOOR COMMUNICATIONS (GLOBCOM 2015).

74. *NYUSIM: The Open Source 5G Channel Model Simulator Software,* N.Y.U. WIRELESS, (Jul. 12, 2016) http://bit.ly/1Snh8YF [https://perma.cc/88TW-P6YE].

75. Hata, *supra* note 67, at 317.

climate type because moisture in the air has a great impact on wave propagation properties.

As mentioned in Section II(E), some of the proposed spectrum bands for 5G networks are difficult to model. The scattering of signals with short wave lengths requires very detailed information about the propagation environment. Propagation models useful for 5G therefore require completely new types of data sources that are more detailed than those currently used. In particular, they will require locations and shapes of buildings, trees (the density of the leaves), lampposts, etc. For example, Google Maps already has a lot of this information available. Google Maps has texturized 3D buildings that are based on Geographic Information Systems data, manually created models, trees positioned and modeled according to imagery from Google Street View, and algorithmically generated 3D models. An alternative is the open source database OpenTopography, which has detailed 3D terrain data based on LiDAR technology, among other things. It is unknown whether Google Maps and OpenTopography are sufficiently developed to be used for real trials.

G. What If . . .

The detailed technical descriptions in the preceding sections were provided to give a sense of the breadth and depth of the spectrum optimization components in the BIA process. Beyond its usefulness in the mechanism of the FCC's BIA, spectrum optimization can be used as a planning tool for regulators to explore repacking of spectrum subject to different requirements. The techniques described in this paper and put in practice in the BIA can be used to efficiently and effectively model the outcomes of various policy decisions, whether national or international, helping regulators make informed decisions on spectrum management issues.

As described earlier, any spectrum requirements (interference protections, protected spectrum bands, etc.) are implemented either in the spectrum optimization as objective functions[76] or as constraints.[77] The choice of implementation should be based on the distinction that constraints *must* be satisfied, while requirements implemented as objective functions are tried to be satisfied to the greatest extent possible. Sections II (B) and (C) gave a whole host of examples of how both types of requirements are implemented by the FCC in the BIA. Using this formulation, in this section, we give a few examples of the type of "what if" scenarios a regulator could consider when tasked with making more spectrum available in a

76. *See infra* Section II (B).
77. *See infra* Section II (C).

band. The output from this scenario analysis would show how much new spectrum the specific scenario could release and specify the quality of the new licenses that could be made available.

When considering national borders, examples of scenarios that could be modeled using the techniques described above are:

- What is the impact on a given allocation if all services in neighboring countries remain allocated to their current frequencies and have their current level of interference protection?
- What are the potential gains if one or more neighbors considers repacking their spectrum at the same time?
- What are the impacts of different types of repacking agreements for different neighbors?

Each of the scenarios would require different levels of cooperation between neighboring countries and there may be very good political reasons why some scenarios are not possible. However, spectrum optimization can be used to simulate different scenarios and use the results to inform decision-makings about the costs and benefits of different approaches.

Regarding incumbents:

- What kinds of solutions are possible if all existing incumbents remain in their current band at current operating parameters?
- What if some incumbents do not need to be repacked in their current band (i.e. being moved to a different band or ceasing to operate)?

Regarding interference between incumbents:

- What is the impact if the allowable interference between licensees were to be increased by some amount (0.5%, 1%, or 2%)?
- What if the interference between incumbents is controlled by a fixed separation distance between the incumbents rather than defined by a propagation model?
- What if a different propagation model is used?
- What if the interference constraints between incumbents are pairwise as in Section II(C)(1), or what if they take account of a full neighborhood of incumbents at the same time?

- What if, in the case of pairwise interference constraints, there is a cap on the aggregate amount of service loss (either coverage or capacity) for each incumbent?

Note that the same questions about interference constraints between incumbents can be asked about interference constraints between incumbents and new users of the spectrum.

The questions above are about choices in design for constraints and objective functions in the spectrum optimization. By investigating the impact of each design option using spectrum optimization, a regulator can make informed decisions about which options yield the best result for the problem they are seeking to solve.

The problem of assigning frequencies to cell sites in a mobile network is also a graph coloring problem, just as assigning channels to TV stations. Mobile operators can consider some of these described approaches to make use of spectrum optimization to study scenarios that achieve optimal coverage in an area. The existing infrastructure could be regarded as incumbents and new potential cells as new users in the framework described in this article.

Because spectrum is a limited resource, regulators can make more spectrum available and users can make better use of newly awarded spectrum licenses by carefully planning networks using optimization techniques.

CONCLUSION

The FCC's Broadcast Incentive Auction has progressed the state of the art in market-based spectrum management, and with the completion of the auction in February 2017, moved the idea from concept to reality.

The BIA created a spectrum optimization approach that incorporates interference elements from different services (broadcast, land mobile, wireless medical telemetry, radio astronomy and commercial mobile) and different countries (US, Canada and Mexico) into an auction mechanism, creating the FCC's first-ever two-sided auction of spectrum, involving both buyers and sellers. While the BIA includes several advances in auction design, one of the least discussed aspects is the role that optimization plays in assessing the voluntary relinquishments of spectrum by broadcasters and repacking the band to include cleared spectrum for new uses.

This article explained the technical underpinnings of the repacking approach and suggested that these techniques have relevance to modern spectrum management challenges beyond a voluntary band-clearing auction. The advanced repacking approaches can be used for strategic planning of spectrum allocations and assignment, allowing regulators and stakeholders to model scenarios that incorporate different radio services, different propagation models and even different national regulations.

With demand for mobile services continuing to grow and with new technologies like 5G just on the horizon, sound and efficient spectrum management is more important now than ever. While the close of the BIA charts the path for this spectrum band, the lessons learned and techniques developed have important global relevance as we chart the digital future.